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General Certificate of Education (A-level)
June 2011

Mathematics MPC3

(Specification 6360)

Pure Core 3

Final

Mark Scheme

Mark schemes are prepared by the Principal Examiner and considered, together with the relevant questions, by a panel of subject teachers. This mark scheme includes any amendments made at the standardisation events which all examiners participate in and is the scheme which was used by them in this examination. The standardisation process ensures that the mark scheme covers the candidates' responses to questions and that every examiner understands and applies it in the same correct way. As preparation for standardisation each examiner analyses a number of candidates' scripts: alternative answers not already covered by the mark scheme are discussed and legislated for. If, after the standardisation process, examiners encounter unusual answers which have not been raised they are required to refer these to the Principal Examiner.

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Key to mark scheme abbreviations

M	mark is for method
m or dM	mark is dependent on one or more M marks and is for method
A	mark is dependent on M or m marks and is for accuracy
B	mark is independent of M or m marks and is for method and accuracy
E	mark is for explanation
√or ft or F	follow through from previous incorrect result
CAO	correct answer only
CSO	correct solution only
AWFW	anything which falls within
AWRT	anything which rounds to
ACF	any correct form
AG	answer given
SC	special case
OE	or equivalent
A2,1	2 or 1 (or 0) accuracy marks
$-x$ EE	deduct x marks for each error
NMS	no method shown
PI	possibly implied
SCA	substantially correct approach
c	candidate
sf	significant figure(s)
dp	decimal place(s)

No Method Shown

Where the question specifically requires a particular method to be used, we must usually see evidence of use of this method for any marks to be awarded.

Where the answer can be reasonably obtained without showing working and it is very unlikely that the correct answer can be obtained by using an incorrect method, we must award **full marks**. However, the obvious penalty to candidates showing no working is that incorrect answers, however close, earn **no marks**.

Where a question asks the candidate to state or write down a result, no method need be shown for full marks.

Where the permitted calculator has functions which reasonably allow the solution of the question directly, the correct answer without working earns **full marks**, unless it is given to less than the degree of accuracy accepted in the mark scheme, when it gains **no marks**.

Otherwise we require evidence of a correct method for any marks to be awarded.

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Q	Solution	Marks	Total	Comments
1 (a)	$\frac{1}{6}$ or $\left(\frac{1}{6}, 0\right)$	B1	1	condone 0.167 AWRT
(b)	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = \frac{1}{x}$	M1		$\frac{k}{x}$ where $k = 1$, 6 or $\frac{1}{6}$
		A1	2	$k = 1$
(c)	$\begin{array}{c\|l} x & y \\ \hline 1 & \ln 6 = 1.7918 \\ 2 & \ln 12 = 2.4849 \\ 3 & \ln 18 = 2.8904 \\ 4 & \ln 24 = 3.1781 \\ 5 & \ln 30 = 3.4012 \\ 6 & \ln 36 = 3.5835 \\ 7 & \ln 42 = 3.7377 \end{array}$	M1		5+ y-values correct, either exact or correct to 3SF (rounded or truncated) or better
		A1		all 7 y-values correct (and only these 7 values), either exact or correct to 3SF (rounded or truncated) or better
	$\mathrm{A} = \frac{1}{3} \times 1\left[(1.7918 + 3.7377) + 4(2.4849 + 3.1781 + 3.5835) + 2(2.8904 + 3.4012)\right]$	M1		correct use of Simpson's rule on their 7 y-values, condone missing square brackets
	$= 18.4$	A1	4	CAO this value only
	Total		**7**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
2(a)(i)	$y = xe^{2x}$			
	$\left(\frac{dy}{dx} =\right) 2xe^{2x} + e^{2x}$	M1 A1 A1 ISW	3	$kxe^{2x} + le^{2x}$ where k and l are 1s or 2s $\left.\begin{matrix} k=2 \\ l=1 \end{matrix}\right\}$ Independent of each other $(= e^{2x}(2x+1))$
(ii)	$x = 1 \Rightarrow \frac{dy}{dx} = 3e^2$	M1		correct substitution of x = 1 into their $\frac{dy}{dx}$ but must have earned M1 in part (i)
	tangent: $y - e^2 = 3e^2(x-1)$ OE	A1	2	CSO (no ISW), must have scored first 4 marks common correct answer: $y = 3e^2x - 2e^2$
(b)	$y = \frac{2\sin 3x}{1+\cos 3x}$			
	$\left(\frac{dy}{dx} =\right) \frac{(1+\cos 3x)6\cos 3x - 2\sin 3x(-3\sin 3x)}{(1+\cos 3x)^2}$	M1		$\frac{\pm p(1+\cos 3x)\cos 3x \pm q\sin 3x(\sin 3x)}{(1+\cos 3x)^2}$ where p and q are rational numbers condone poor use/omission of brackets PI by further working
	$= \frac{6\cos 3x + 6\cos^2 3x + 6\sin^2 3x}{(1+\cos 3x)^2}$	A1		this line must be seen in this form (ie in terms of $\cos^2 3x$ and $\sin^2 3x$), but allow $\sin^2 3x$ replaced by $1-\cos^2 3x$ condone denominator correctly expanded
		m1		correct use of $k\sin^2 3x + k\cos^2 3x = k$ or $k\sin^2 3x = k(1-\cos^2 3x)$
	$= \frac{6\cos 3x + 6}{(1+\cos 3x)^2}$			
	$= \frac{6}{1+\cos 3x}$	A1	4	CSO
	Total		**9**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
	note: if degrees used then no marks in (a) and (c)			
3(a)	$\mathrm{f}(x) = \cos^{-1}(2x-1) - \mathrm{e}^x$			or reverse
	$\left.\begin{array}{l}\mathrm{f}(0.4) = 0.3\\ \mathrm{f}(0.5) = -0.1\end{array}\right\}$	M1		sight of ± 0.3 (AWRT) **AND** ∓ 0.1 (AWRT)
	change of sign $\therefore 0.4 < \alpha < 0.5$	A1	2	CSO, note f (x) must be defined, condone $0.4 \le \alpha \le 0.5$
		(M1)		alternative method $\left.\begin{array}{l}\mathrm{e}^{0.4} = 1.5,\ \cos^{-1}(2\times0.4 - 1) = 1.8\\ \mathrm{e}^{0.5} = 1.65,\ \cos^{-1}(2\times0.5 - 1) = 1.57\end{array}\right\}$
		(A1)		$\left.\begin{array}{l}\text{at } 0.4\ \mathrm{e}^x < \cos^{-1}(2x-1)\\ \text{at } 0.5\ \mathrm{e}^x > \cos^{-1}(2x-1)\\ \therefore 0.4 < \alpha < 0.5\end{array}\right\}$
(b)	$\cos^{-1}(2x-1) = \mathrm{e}^x$ $2x - 1 = \cos(\mathrm{e}^x)$ $x = \frac{1}{2}\left(\cos(\mathrm{e}^x) + 1\right) = \frac{1}{2} + \frac{1}{2}\cos(\mathrm{e}^x)$	B1	1	AG must see middle line, and no errors seen, but condone cos e^x
(c)	$x_1 = 0.4$			
	$x_2 = 0.539$	B1		CAO
	$x_3 = 0.428$	B1	2	CAO
	Total		**5**	

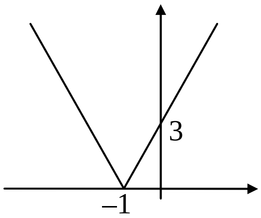
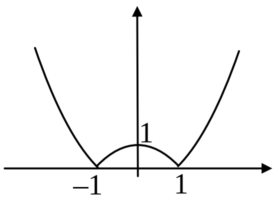
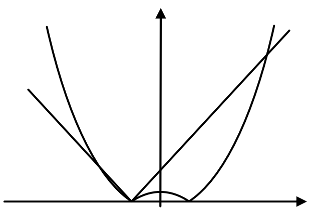
MPC3 (cont)

Q	Solution	Marks	Total	Comments
4(a)(i)	$\left(\sin^{-1} \pm 0.25 =\right) \pm 14.5$	M1		PI by sight of 194.5 etc condone ± 14.4
	$\theta = 194.5,\ 345.5$ (AWRT)	A1	2	no extras in interval, ignore answers outside interval
(ii)	$2\cot^2(2x+30) = 2 - 7\operatorname{cosec}(2x+30)$			condone replacing 2x + 30 by Y
	$2(\operatorname{cosec}^2(2x+30) - 1) = 2 - 7\operatorname{cosec}(2x+30)$	M1		correct use of $\operatorname{cosec}^2 Y = 1 + \cot^2 Y$
	$2\operatorname{cosec}^2(2x+30) + 7\operatorname{cosec}(2x+30) - 4 (= 0)$	A1		must be in this form
	$(2\operatorname{cosec}(2x+30) \pm 1)(\operatorname{cosec}(2x+30) \pm 4)(= 0)$	m1		attempt at factorisation
	$\operatorname{cosec}(2x+30) = \frac{1}{2}$ or -4	A1		must be this line using f (2x + 30)
	$2x + 30 = 194.5,\ 345.5$			
	$x = 82.2,\ 157.8$ (AWRT)	B1		one correct answer, allow 82.3, ignore extra solutions
		B1	6	CAO both answers correct and no extras in interval, ignore answers outside interval
(b)	stretch (I) scale factor $\frac{1}{2}$ (II) parallel to x-axis (III)	M1 A1		I and either II or III I + II + III
	translate	E1		
	$\begin{pmatrix} -15 \\ 0 \end{pmatrix}$	B1	4	condone '15 to left' or '–15 in x (direction)'
	alternative method translate	(E1)		
	$\begin{pmatrix} -30 \\ 0 \end{pmatrix}$	(B1)		
	stretch	(M1)		as above
	scale factor $\frac{1}{2}$ parallel to x-axis	(A1)		as above
	Total		**12**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
5(a)	$\left[\mathrm{f}(x)\right]$ not $1-1$	E1	1	OE
(b)	$y=\dfrac{1}{2x+1}$			
	$x=\dfrac{1}{2y+1}$	M1		swap x and y } either order
	$2y+1=\dfrac{1}{x}$	M1		a correct next line } either order
	$\left[g^{-1}(x)=\right]\dfrac{1}{2}\left(\dfrac{1}{x}-1\right)$ OE	A1	3	$[y=]\dfrac{1}{2}\left(\dfrac{1}{x}-1\right)$
(c)	$\left[\mathrm{g}^{-1}(x)\right]\neq -0.5$	B1	1	sight of $\neq -0.5$ OE
(d)	$\left(\dfrac{1}{2x+1}\right)^2=\dfrac{1}{2x+1}$	B1		sight of $\left(\dfrac{1}{2x+1}\right)^2$ or $\dfrac{1}{(2x+1)^2}$
	$(2x+1)=(2x+1)^2$ or $2x+1=4x^2+4x+1$ or $\dfrac{1}{2x+1}=1$ or $2x+1=1$	M1		one correct step, must be one of these four lines
	$x=0$	A1	3	CSO
	Total		**8**	
6(a)	$3\ln x=4$ $\left(\ln x=\dfrac{4}{3}\right)$			
	$x=\mathrm{e}^{\frac{4}{3}}$	B1	1	ISW. Condone $\sqrt[3]{\mathrm{e}^4}$
(b)	$3\ln x+\dfrac{20}{\ln x}=19$			
	$3(\ln x)^2+20=19\ln x$	M1		correctly multiplying by $\ln x$.
	$3(\ln x)^2-19\ln x+20(=0)$	A1		
	$(3\ln x\pm 4)(\ln x\pm 5)(=0)$	m1		use of formula, or completing the square must be correct
	$\ln x=\dfrac{4}{3},\ 5$	A1		
	$x=\mathrm{e}^{\frac{4}{3}},\ \mathrm{e}^5$	A1	5	condone $\sqrt[3]{\mathrm{e}^4}$
	Total		**6**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
7(a)(i)		M1		modulus graph, approximate V shape, touching negative x-axis and crossing y-axis
		A1	2	-1, 3 marked, graph symmetrical, straight lines
(ii)		M1		modulus graph in 3 sections, touching x-axis and crossing positive y-axis
		A1		correct curvature; their $x>1$, their $x<-1$ } independent
		A1	3	correct curve $-1 \le x \le 1$ and $x = \pm1$, $y = 1$ marked } independent
(b)(i)	$\lvert 3x+3 \rvert = \lvert x^2-1 \rvert$ $(3x+3 = x^2-1)$ $(0 =)\ x^2-3x-4$ —A	M1		either A or B seen, all terms on one side
	$x = 4, -1$	A1,A1		
	$(3x+3 = 1-x^2)$ $x^2+3x+2\ (=0)$ —B $x = -1, -2$	A1,A1	5	$\therefore\ x = -2, -1, 4$ SC NMS or partial method 1 correct value 1/5 2 correct values 2/5 } independent of method mark 3 correct values 5/5 more than 3 distinct values max 2/5
(ii)	 $x > 4,\ \ x < -2$	M1,A1	2	$x >$ their largest, $x <$ their smallest; CAO
	Total		**12**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
8	$\int \frac{1}{\cos^2 x(1+2\tan x)^2}\,dx$			
	$u = 1 + 2\tan x$			
	$\left(\frac{du}{dx} =\right) 2\sec^2 x$ OE	M1		condone $\left(\frac{du}{dx} =\right) a\sec^2 x$ where a is a constant
	$\int = \int \frac{du}{2u^2}$	m1		$\int \frac{k}{u^2}\,(du)$, where k is a constant
		A1		correct, or $\frac{1}{2}\int u^{-2}\,(du)$
	$= \frac{1}{2}\frac{u^{-1}}{-1}$	A1F		correct integral of their expression but must have scored M1 m1
	$= -\frac{1}{2u}$			
	$= -\frac{1}{2(1+2\tan x)}(+c)$	A1	5	CSO, no ISW
	Total		**5**	

MPC3 (cont)

Q	Solution	Marks	Total	Comments
9 (a)	$\int x \ln x \, \mathrm{d}x$			
	$u = \ln x \quad \frac{\mathrm{d}v}{(\mathrm{d}x)} = x$; $\frac{\mathrm{d}u}{(\mathrm{d}x)} = \frac{1}{x} \quad v = \frac{x^2}{2}$	M1		correct direction and sight of $\frac{1}{x}, \frac{x^2}{2}$
	$\int = \frac{x^2}{2}\ln x - \int \frac{x^2}{2} \times \frac{1}{x} \, (\mathrm{d}x)$	A1		
	$= \frac{x^2}{2}\ln x - \frac{x^2}{4} (+c)$	A1	3	
(b)	$y = (\ln x)^2$			
	$\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right) = 2\ln x \times \frac{1}{x}$	M1		$\frac{k}{x}\ln x$ where $k = \frac{1}{2}, 1$ or 2
		A1	2	$k = 2$
(c)	$y = \sqrt{x}\,\ln x$			
	$(V =) \pi \int_1^{\mathrm{e}} x(\ln x)^2 \, \mathrm{d}x$	B1		all correct, incl brackets, π, limits and dx (but dx may be seen BEFORE this line)
	$u = (\ln x)^2 \quad \frac{\mathrm{d}v}{(\mathrm{d}x)} = x$; $\frac{\mathrm{d}u}{(\mathrm{d}x)} = 2\ln x \frac{1}{x} \quad v = \frac{x^2}{2}$	M1		correct direction with $\frac{\mathrm{d}u}{(\mathrm{d}x)} = \frac{k}{x}\ln x$ where $k = \frac{1}{2}, 1$ or 2 and sight of $\frac{x^2}{2}$
	$\int = \frac{x^2}{2}(\ln x)^2 - \int \frac{x^2}{2} \times \frac{2}{x} \ln x \, (\mathrm{d}x)$	m1		correct substitution of their terms into the parts formula
	$= \frac{x^2}{2}(\ln x)^2 - \int x \ln x \, (\mathrm{d}x)$	A1		integral needs to be simplified to $\int x \ln x$
	$= \frac{x^2}{2}(\ln x)^2 - \frac{1}{4}x^2(2\ln x - 1)$ OE			
	$V = (\pi)\left[\frac{x^2}{2}(\ln x)^2 - \frac{1}{4}x^2(2\ln x - 1)\right]_1^{\mathrm{e}}$			
	$= (\pi)\left[\left(\frac{\mathrm{e}^2}{2} - \frac{1}{4}\mathrm{e}^2\right) - \left(0 + \frac{1}{4}\right)\right]$	m1		correct substitution of 1 and e into their expressions of the form $px^2(\ln x)^2 + qx^2\ln x + rx^2$ where p, q and r are non-zero rational numbers, and an intention to subtract. Do not condone F(1) – F(e)
	$= \frac{\pi}{4}\left[\mathrm{e}^2 - 1\right]$ **OE**	A1	6	$\pi\left[\frac{\mathrm{e}^2}{4} - \frac{1}{4}\right]$ etc
	Total		**11**	
	TOTAL		**75**	